A Rule with Diagonal Scales for Sines & Chords[1]

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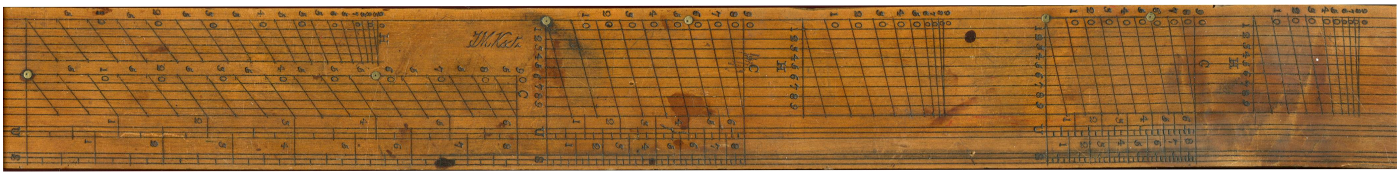
Introduction
In the Netherlands a number of special wooden rules are known to exist, with a remarkable layout of scales. The most striking aspects of these rules are multiple diagonal scales, and the fact that most of the diagonal scales represent trigonometric functions. As example this article will describe one of these, a boxwood specimen signed by Kleman, a Dutch family of instrument makers in Amsterdam from late 18th century to mid 19th century.

Description of the rule
The dimensions of this particular rule are 343 x 35 x 5 mm.
The front side (with bevelled upper edge) bears a regular diagonal scale, in units of 2.2 cm from right to left; the diagonals at the left side allow measurements in units of 4.4 cm. Diagonal scales for geodesy or cartography usually have a scale factor inscribed to allow using the diagonals with certain map scales, but not here. The diagonal scale allows a distance to be measured with the precision of one additional (third) decimal, to be read on the horizontal intersecting the diagonal, see [1].

The back side contains the trigonometric scales. There are three sets of two scales each, one abbreviated "H" (for *Hoeckmaet*) and the other "C" (for *Coorde*). Up into the 19th century these old Dutch names were used for the Sine and Chord function respectively (the chord is two times the sine of half the angle).



Scale details
For clarity the next figure shows only the enlarged leftmost set of "H" and "C" diagonals. Along the upper edge of the "H" diagonals the sines of the angle values 0° through 90° are plotted in increments of 5°. The diagonals increase the precision to 1°. The distance between 0° and a given angle is the sine value of that angle in proportion to the full length of 0° to 90°. This full length, the *radius* value, is for each of the three diagonal sets 7½, 3, and 2¼ cm respectively, from left to right. For example at 30° one finds on the 7½ cm sine-scale a distance of 7½ : 2 = 3¾ cm from 0°, because sin(30°) = 0.5.

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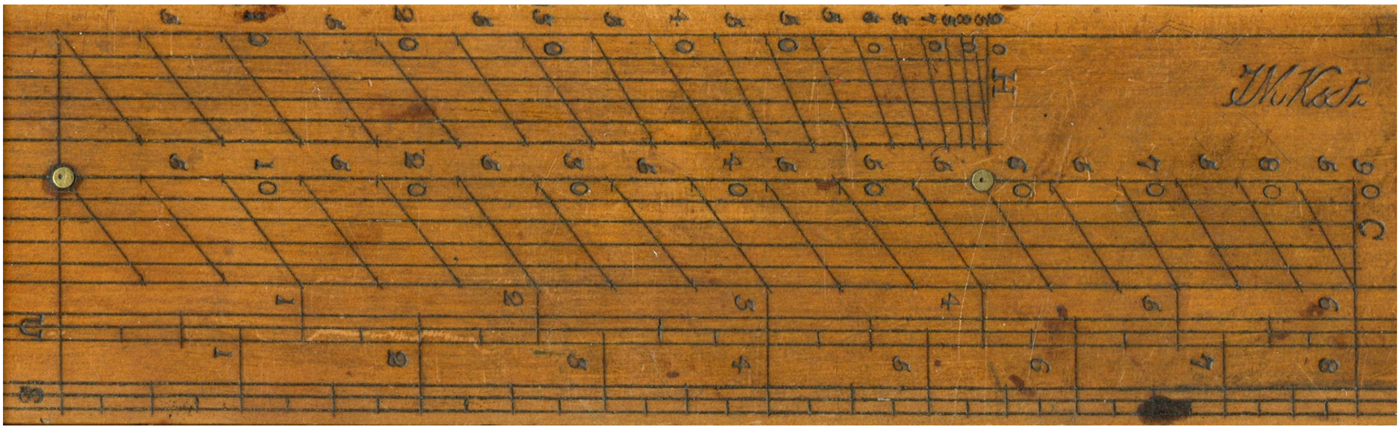

In each of the three diagonal sets the transversals of the chords (C-scale) follow the same principle, but now the radius-value is at 60°. The chord value of 90° is √2 times the radius.
In each chord scale one finds two brass inserts, at 0° and at 60° respectively, to guide the points of the dividers and to protect the wood against the sharpness of those points.

It may seem remarkable that the non-linear sine and chord functions are approximated by straight diagonals, but the error introduced by that approximation is nowhere greater than 0.5 %. Anyhow it would have been more difficult for the instrument engravers to draw with their burins other lines than straight ones or arcs of circles.
Along the lower edge of the sine and chord diagonals one finds two angle scales in alternative units: “U” voor *uuren* (hours) on a 24-hour dial (where 90° is equal to 6 hours) and “S” voor kompas-*streeken* (compass points or *rhumbs*). One quarter of a ship’s compass rose is divided in 8 points, for example from North to East: N,N by E, NNE, NE by N, NE, NE by E, ENE, E by N, E. The units of these two scales suggest usage of the rule for sea navigation and/or sundials.

Application
The challenge of understanding this rule is the following: all scales are mathematically correct and clear; but how would a user have applied these sets of sine & chord scales, with their diagonals and the three different radii? And in what application or profession?
The rule is not a goniometrical *table*, meaning that it is not intended to calculate a numerical value of a sine or chord function at a given angle: there is no linear scale along the sine & chord transversals from which to read a function value!
Actually the only linear scale on this rule is the regular diagonal scale at the front, and that scale is based on units of 2.2 cm: not related to the radii on the back side, nor to any Dutch inch-like measure such as “Amsterdamsche duimen” or “Rijnlandsche duimen”. In the Netherlands no thumb-based (“duimen”) length unit was known as small as 2.2 or as large as 4.4 cm.

Plain-Scale
The answer is probably found in the similarity with the “plain-scale” (in Dutch a *plein-schael*), a sea navigation rule that was known already before the invention of the logarithm. A typical 17th century plain-scale contained scales such as the pair CHORD and LON (LONgitude or “Miles of Longitude”): this scale pair allowed the calculation of the number of nautical miles in one degree longitude (LON) along a parallel at a given latitude (marked on the CHORD scale). Often there were additional scales on a plain-scale: the non-logarithmic front side of a common Gunter rule actually is an extensive plain-scale, see [2]. Both chord and sine scales are included on most plain-scales. Two well-known plain-scales (one owned by the Rijksmuseum, the other by the Scheepvaartmuseum, both in Amsterdam) contain even one same set of sine & chord diagonals as the three sets in our Kleman rule.

Usage
Descriptions of the usage of plain-scales do exist, though few in number and often wanting in completeness. The plain-scale was mainly used to draw graphical constructions with a set of dividers on a sheet of paper to solve navigational problems, the prime example being the course triangle with wind and/or current components, see [3]. For this purpose the chord scale was used to set or to measure angles in the triangle, while the linear diagonal scale quantified the lengths of the triangle’s sides; on some of our specimens the linear transversal scale was named for this reason “M”, for

“mijlen” (miles). There was no relation with map scales as the constructional drawings were made on blank paper: the seemingly arbitrary choice of the unit 2.2 cm on the front was probably based on the fact that any number from 000 to 100 could fit on a regular sheet of paper. The rule was “unit-less” because arbitrary distance units could be represented such as leagues, nautical miles or other.
Any relation between the three angles and the three sides in a rectangular or oblique triangle could be calculated by constructing and measuring the triangle on paper.

Note that the sine function was not needed for these triangle calculations as the construction method did not use the sine rule. The only known example of using the sine scale is the construction of an orthographical projection of the globe, see [4]. The orthographical projection shows the globe projected from a point at infinite distance. Sine scale values were used to set off the positions of parallels of given latitude from the equator. We do not know why a navigator would need to draw an orthogonal projection as a regular routine, except for spherical calculations. It appears that the chord scale must have been of much more practical use than the sine scale.

The reason for the three different radius values in the diagonals turns out to be very practical. The navigator wanted to size his drawing on the available space on his paper, but also depending on the complexity of his drawing. In [3] some complex examples of spherical calculations by construction are given, requiring smaller circles within large circles. It is also possible to improve the readability of drawings with more than one angle in a single point by using different radii for different angles.
The engraved alternative units for angles, in *rhumbs* and *hours*, are useful for calculations related to compass courses and sun measurements for time determination respectively.

Attribution
De maker of this boxwood rule was J. M. (Jan Marten) Kleman & Zn., see the abbreviation *J M K & Zn* in de upper right corner of the enlarged picture above. Kleman was an instrumentmakers firm in Amsterdam with a history from 1781 to 1859. Until 1809 products were signed “J.M. Kleman”, after that date the signature became “J.M. Kleman & Zoon”. This means that the rule can be dated to the period between 1809 and 1859.

Similar rules in the Netherlands
In the Netherlands, five other similar rules are known at private collections, antiquarians, and the Museum Boerhaave in Leiden (inventory nr. V07645): one with an identical signature of Kleman, one signed by Gerard Hulst van Keulen (from the well-known house Hulst van Keulen, active from late 17th century to 1885 in cartography and navigational instruments), and three unsigned specimens.
Each of the six similar rules uses the same radius values 7½, 3 and 2¼ cm, and a base measure of 2.2 cm on the diagonal scale at the front; the total dimensions of the rule bodies however vary, their lengths ranging from about 30 to 40 cm. All of the six rules are of Dutch origin, given the signatures on some of them, and more inportantly the *Dutch* scale abbreviations on all of them. The rule by Gerard Hulst van Keulen dates from end 18th century, the others probably from the 19th century.
The rather large number of 6 known rules of this type suggests that usage by Dutch navigators must have enjoyed at least some popularity.
It is not yet known whether rules of this type have also been found in other countries.

References

1. Poelje, O.E van, *Diagonals and Transversals: Magnifying the Scale*, Journal of the Oughtred Society, Vol. 13:2, 2004, page 22.
2. Poelje, O.E van, *Gunter Rules in Navigation*, Journal of the Oughtred Society, Vol. 13:1, 2004, page 11.
3. Ruyter, Dierick, *De Platte ofte Pleyn-schael verclaert*, Middelburgh, 1631.
4. Heather, J. F., *A Treatise on Mathematical Instruments*, London, 1853, page 18.

Questions to the reader:

Who can give some more information on the usage of this type of rule?
Have such rules with trig scale diagonals also been found in other countries than the Netherlands?